

# Some problems in plasma suppression of beam-beam interactions at muon colliders. \*

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## Abstract

The idea of plasma suppression of beam-beam effects at muon colliders is discussed. It is shown that one should take into account collisions in the plasma that were ignored before. Rough estimates show that this effect leads to a fast “recovery” of the beam magnetic field. For beam parameters characteristic for muon colliders the suppression of the magnetic component of the beam field (1/2 of the total force) is almost absent. It is also shown that the presence of the dense plasma (Li jet) at the interaction point leads to enormous hadronic background (due to photo-nuclear reactions) in the detector, about  $10^7$  particles per crossing at large angles which creates serious problems for experimentation.

## 1 Introduction

One of main the problems for high energy muon colliders is the limitation of the luminosity due to beam-beam interactions. A measure of the beam-beam interaction is the tune-shift parameter  $\xi$  [1]. For round beams,  $\xi = Nr_c/4\pi\epsilon_n$ , where  $r_c = e^2/mc^2$  is the classical radius of the beam particles,  $\epsilon_n$  is the normalized transverse emittance. The value of  $\xi$  should be small enough ( $\xi_{max} < 0.1$ ) - otherwise the beams are disrupted due to resonance diffusion. The maximum luminosity  $L_{max} = N\gamma f\xi_{max}/r_c\beta$ , where  $\beta$  is the  $\beta$ -function (usually  $\beta \approx \sigma_z$ ),  $f$  is the collision rate. So,  $L \propto \xi$ . This effect puts a severe limit on the luminosity of muon colliders.

One of the possible solution of this problem is plasma suppression of beam-beam interactions [2, 3, 4]. In a sufficiently dense plasma one can expect that the induced charges and currents will decrease the beam fields and, consequently, the beam-beam effects. If plasma decreases the beam field by a factor K, one can use beams with K

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times smaller  $\epsilon_n$  (to keep  $\xi = \xi_{max}$ ), correspondingly the luminosity will be K times larger.

It is essential to reduce both electric and magnetic fields because in the vacuum their action on the opposing beam are equal in value and direction, the effective beam field is  $|E| + |B|$ . If the plasma density  $n_p$  is larger than the particle density of the colliding bunches  $n_b$ , the electric field of the beam will be suppressed by repelling (in the case of negatively charged bunch) or attracting (in the case of positively charge bunch) plasma electrons (ions are immobile). The nature of the magnetic field suppression is somewhat more complicated. In the linear approximation the resulting suppression of the beam-beam interaction [3]

$$\xi/\xi_0 \sim 4/(k_p\sigma_r)^2 \text{ for } k_p\sigma_r \gg 1, \quad (1)$$

where  $k_p = \omega_p/c$ ,  $\omega_p^2 = 4\pi n_p e^2/m_e$ ,  $\sigma_r$  is the r.m.s. beam radius. A more accurate result, including nonlinear effects and finite plasma thickness, was obtained in ref. [4].

So, for a factor of 5 suppression of beam-beam interactions the plasma should satisfy the following requirements: a)  $n_p > n_b$  and b)  $k_p\sigma_r > 4$ . For example, let us take parameters of the “evolutionary” 100 TeV muon collider (see the B.King’s table) but with 5 times smaller  $\epsilon_{nx}$ :  $N = 0.8 \times 10^{12}$ ,  $\sigma_r = \sqrt{2}\sigma_x = 0.13 \mu\text{m}$ ,  $\sigma_z = 0.25 \text{ cm}$ . In this case  $\xi = 0.5$  without suppression, while the acceptable  $\xi = 0.1$ . The plasma density required for decreasing  $\xi$  by a factor of 5 is found from conditions a) and b), which give  $n_p > 2.3 \times 10^{21}$  and  $n_p > 2.5 \times 10^{22}$ , respectively. As a source of plasma one can use a liquid Li jet [4] with electron density  $1.5 \times 10^{23} \text{ cm}^{-3}$ . Such a target will be fully ionized by the muon beam and the return current. If this theory is correct, in the considered example one can increase the luminosity by a factor of 5. With other beam parameters one can expect even better results, up to 30 for the given beam diameter. All this sounds nice. However, there are two effects which create serious problems for this method, and, perhaps, close it:

- collisions in the plasma;
- hadronic background at large angles.

## 2 Collisions in the plasma.

In all papers on plasma suppression of the beam-beam interaction it was assumed that plasma is collisionless. This picture is not correct. In fully ionized plasma the electrons of the return current do not lose energy on ionization, do not lose energy in collisions with other electrons, because all electrons move with the same average velocity, and have very small energy loss in collisions with the ions; however, their longitudinal velocity is decreased due to the scattering on the ions.

The change of the longitudinal velocity in one scattering on the ion

$$\Delta v = -2v_0 \sin^2 \frac{\vartheta}{2} \approx -v_0 \frac{\vartheta^2}{2} \quad (2)$$

The resulting friction in the plasma

$$\begin{aligned}\frac{d\vec{v}}{dt} &= - \int v_0 \Delta v n_p d\sigma = - \int v_0^2 \frac{\vartheta^2}{2} 8\pi n_p \left( \frac{e^2 Z}{m_e v_0^2} \right)^2 \frac{d\vartheta}{\vartheta^3} = \\ &= -4\pi n_p \frac{\vec{v}}{v_0} \left( \frac{e^2 Z}{m_e v_0} \right)^2 \ln \frac{\vartheta_{max}}{\vartheta_{min}} = -4\pi n_p \frac{\vec{v}}{v_0} \left( \frac{e^2 Z}{m_e v_0} \right)^2 \ln \Lambda,\end{aligned}\quad (3)$$

where  $\ln \Lambda = \ln(b_{max}/b_{min})$ . The minimum value of the impact parameter  $b$  follows from the energy conservation  $b_{min} = e^2 Z / m v_0^2$ , and the maximum value of  $b$  is equal to the Debye length  $\lambda_D = \sqrt{kT/m}/\omega_p$ . For the considered plasma densities and electron velocities,  $\ln \Lambda \approx 7 \sim 10$ .

For estimation of the collision time one can take  $dv = v$ , which gives

$$\tau_{col} \sim \frac{v_0}{4\pi n_p \left( \frac{e^2 Z}{m_e v_0} \right)^2 \ln \Lambda}.\quad (4)$$

The average velocity of electron in the return current  $u \approx (n_b/n_p)c$ . Although this velocity is not the same as  $v_0$  due to transverse motion, let us the first assume  $v_0 = u$ . Then

$$\tau_{col} \sim \left( \frac{n_b}{n_p} \right)^3 \frac{1}{4\pi c n_p Z^2 r_e^2 \ln \Lambda}.\quad (5)$$

For the example given above  $n_b/n_p \sim 1/60$ ,  $Z = 3$ ,  $n_p = 1.5 \times 10^{23}$ , that gives  $\tau_{col} \sim 10^{-17}$  sec, which is much smaller than the bunch collision time  $\sigma_z/c \sim 0.25/3 \times 10^{10} \sim 10^{-11}$  sec.

So, the assumption of collisionless plasma is not valid. The plasma should be considered as a medium with some conductivity  $\sigma_c$ . Accurate calculation of conductivity is a complicated task because the electron drift in the longitudinal induction electric field with very small energy loss, only scatter. Due to the field their total kinetic energy continuously grows, while the drift velocity is approximately constant:  $u \sim c(n_b/n_p)$ . Nevertheless, we can make some estimate.

Eq.3 is approximately valid even in the case of the “hot” return current if we will consider  $\vec{v}$  as the drift velocity (from now on  $u$ ) and  $v_0$  as the “thermal” velocity, which is still unknown. The loss of the drift velocity given by Eq.3 is compensated by the induction electric field

$$|du/dt| = eE_{||}/m_e.\quad (6)$$

The conductivity is defined by equation

$$en_p u = \sigma_c E.\quad (7)$$

Drift velocity is known:

$$u = c(n_b/n_p).\quad (8)$$

Using Eqs 6,7,8, we get

$$\sigma_c = \frac{e^2 n_b c}{m_e |du/dt|} \quad (9)$$

Using Eq.3, we obtain

$$\sigma_c = \frac{m_e v_0^3}{4\pi e^2 Z^2 \ln \Lambda} \quad (10)$$

Now we have to estimate  $v_0$ . The collision time is given by eq.4. Between two collisions the electron drift velocity is restored by the induction electric field and the total energy is increased by about  $m_e u^2$ , so as an estimate one can take the average kinetic energy to be equal to  $m_e v_0^2 \sim N_{col} m_e u^2$ . The maximum number of scatterings for the same electron can be estimated as the number of collisions after which its transverse displacement is equal to the beam radius (then this electron in the return current is replaced by the new one which comes from outside and is initially cool)

$$\tau_{col} v_0 \sqrt{N} \sim \sigma_r. \quad (11)$$

Using this arguments and Eq.10 we find

$$v_0 = c(4\pi n_b Z^2 \sigma_r r_e^2 \ln \Lambda)^{1/5} \quad (12)$$

For  $Z=3$  (Li),  $n_b = 2.5 \times 10^{21}$  (see the example above) and  $\sigma_r = 0.15 \mu\text{m}$ ,  $v_0 \sim 0.075c$ .

So, the conductivity is found, see Eqs 10,12. Now we have to understand how the conductivity influences the plasma suppression of the bunch field. The return current is driven by the longitudinal electric field  $E_{||}$  that is caused by penetration of the beam magnetic field into the plasma. From Faraday law

$$E_{||} \sim \frac{1}{c} \frac{d(B_\phi \sigma_r)}{dt}. \quad (13)$$

This electric field produces the return current equal approximately to the bunch current  $I$  (if compensation works)

$$\sigma_c E_{||} \pi \sigma_r^2 \sim I. \quad (14)$$

Introducing  $B_\phi \sim 2I/c\sigma_r$  (the beam field when there is no beam field suppression) and Eqs 13,14, we obtain

$$\frac{dB_\phi}{B_\phi} \sim \frac{c^2 dt}{2\pi \sigma_r^2 \sigma_c}. \quad (15)$$

The relative value of the beam field which penetrates into the plasma during the time of the bunch collision is

$$\frac{\Delta B}{B} \sim \frac{c\sigma_z}{2\pi \sigma_r^2 \sigma_c} \sim \frac{2\sigma_z r_e Z^2 \ln \Lambda}{\sigma_r^2 (v_0/c)^3}, \quad (16)$$

where  $v_0$  is given by Eq.12. For the example considered in this paper:  $\sigma_z = 0.25$  cm,  $Z = 3$ ,  $\sigma_r = 0.13$   $\mu\text{m}$ ,  $N = 0.8 \times 10^{12}$ ,  $v_0/c \sim 0.075$ ,  $\ln \Lambda = 7$  we get

$$\frac{\Delta B}{B} \sim 100 !!! \quad (17)$$

In order to obtain plasma suppression by one order of magnitude we need  $\Delta B/B \sim 0.1$ . So, it seems that plasma does not help. Although my estimate is very approximate, it is very unlikely that a factor of 1000 is lost.

## 3 Backgrounds

### 3.1 Photo-nuclear reactions

The total photo-nuclear cross section for lithium is [5]

$$\sigma_{\gamma Li} \sim 0.4 \times 10^{-27} \text{ cm}^2. \quad (18)$$

The number of virtual photon with the energy above 1 GeV per one muon at 100 TeV muon collider is

$$N_\gamma \sim \int \frac{2\alpha}{\pi} \ln \left( \frac{E}{\omega} \right) \frac{d\omega}{\omega} \sim 0.3 N_\mu. \quad (19)$$

The number of ph.n. reactions per bunch crossing generated by  $2 \cdot 10^{12}$  muons (two beams) in  $l = 0.5$  cm Li jet is

$$N_b = N_\gamma n_{Li} l \sigma_\gamma \sim 0.3 \times 2 \cdot 10^{12} \times 5 \cdot 10^{22} \times 0.5 \times 0.4 \cdot 10^{-27} = 0.6 \cdot 10^7 ! \quad (20)$$

Although most of the produced particles travel in the forward direction, each reaction produce gives approximately one particle ( $\pi^\pm, \pi^0$ ) at large angles, with  $P \sim P_t \sim 300$  MeV. The total energy of these particles is greater than  $2 \cdot 10^3$  TeV.

It is hard to imagine a detector which could work in conditions so terrible!

### 3.2 $e^+e^-$ production: $\mu Li \rightarrow \mu Li e^+e^-$

The cross section of this reaction [6]

$$\sigma \approx \frac{28\alpha^2 r_e^2}{27\pi} (l^3 - 6.36l^2)(Z_1 Z_2)^2, \quad (21)$$

where  $l = \ln \frac{2(P_1 P_2)}{m_1 m_2} \approx \ln 2\gamma_\mu$ . In the case of 50 TeV muons and the Li target,  $\sigma = 1.8 \times 10^{-26} \text{ cm}^2$

The probability of  $e^+e^-$  pair creation by a muon in a 1 cm thick Li jet for 1000 crossings (as it is in the muon colliders) is about  $1 - e^{-1}$  (one interaction length). In most cases the energy loss is not large but sufficient to knock the muon out of the  $10^{-4}$  energy range which contributes to luminosity (muons with larger energy deviations are defocussed due to chromatic aberrations of the final focus system). So, this effect will decrease the luminosity lifetime by about a factor of 2.

## 4 Conclusions

Suppression of the beam-beam effects by a dense plasma jet (Li) at the collision point is a very attractive idea. However, collisions in the plasma significantly change the picture. This effect was ignored before. Rough estimates show that this effect leads to fast “recovery” of the magnetic beam field, leaving it practically unsuppressed. This result should be checked by more accurate calculations.

Photo-nuclear reactions produce enormous hadronic backgrounds in the detector ( $\sim 10^7$  particles/crossing at large angles), so the possibility of experimentation at such background conditions is practically impossible. Electro-production of  $e^+e^-$  pairs in Li jet leads to some decrease of the luminosity lifetime.

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